# An analytic expression for the HBT extrinsic base-collector capacitance derived from S-parameter measurements

E. Wasige, B. Sheinman, V. Sidorov, S. Cohen, and D. Ritter
Department of Electrical Engineering, Technion - Israel Institute of Technology, 32000 Haifa. ISRAEL.

*Abstract*— **Direct extraction is the most accurate method for the determination of equivalent-circuits of heterojunction bipolar transistors (HBTs). However, previous work lacks an exact expression for the extrinsic base-collector capacitance, which models the distributed nature of the base. This paper gives the derivation of an exact expression for this capacitance. As a result, each intrinsic equivalent-circuit parameter is determined using a simple exact expression at each measured frequency. The expression is valid for both the hybrid-$\pi$ and the physics-based T-topology equivalent circuits. Extraction results for InP- and GaAs-HBTs are given. Note that a method for accurately extracting the sum of the extrinsic and intrinsic base-collector capacitances exists [6].**

## I. Introduction

In the analysis and design of microwave and millimeterwave circuits using HBTs, it is necessary to have an accurate linear equivalent circuit. The last decade has seen an increasing shift from the traditional optimisation techniques to the more accurate and simpler direct extraction techniques (see e.g. [1], [7]- [10] for the T-topology, and [11]-[16] for the hybrid-$\pi$ equivalent circuits).

This approach greatly simplifies parameter extraction since methods already exist for determining the bias-independent extrinsic part of the HBT, which comprises pad capacitances, access resistances and inductances [3]- [5], [11]. These so-called parasitic elements can be determined from an analysis of measured S-parameters of either special test structures [11], [6], or measurements at specific bias points (cut-off mode, open collector, and/or forward bias) [3], [5]. The accuracy of these methods is best tested with the exact formulation for the intrinsic device proposed in this paper.

The direct extraction method is in analogy to the method developed for FETs by Dambrine [2]. However, it has proved difficult in HBT modeling to resolve the seven (7) intrinsic elements (namely $R_{bi}$, $C_{bcx}$, $R_{be}$, $C_{be}$, $R_{bc}$, $C_{bci}$, and $\alpha$ for the T-topology, or $R_{bi}$, $C_{bcx}$, $R_\pi$, $C_\pi$, $R_{bc}$, $C_{bci}$, and $G_m$ for the hybrid-$\pi$ equivalent circuits (see Fig.1 and 2 )) in terms of the four (4) measured complex S-parameters, the main difficulty being with the base-collector capacitance $C_{bcx}$, which models the distributed nature of the base. Conventional analysis of the intrinsic circuit leads to a complicated interrelationship of the elements, especially due to $C_{bcx}$(see Fig.1) which bridges the intrinsic network. Often, as a result, some simplifying assumptions or special extra measurements had to be made (see e.g. [8], [11]- [16]), or optimisation steps had to be used [3], [4].

Employing the de-embedding technique, which is normally used on known parasitics, on $C_{bcx}$, this paper gives the derivation of an exact expression relating this base-collector capacitance to the measured S-parameters (de-embedded of parasitic elements and converted to Y-parameters). Consequently, all the intrinsic parameters are resolved uniquely in terms of Z- or Y-parameters at each measured frequency using simple exact equations.

The ratio between the extrinsic and intrinsic capacitances is given as (emitter mesa area)/(base mesa area - emitter mesa area). A convenient and practical method used to determine the individual base-collector capacitances, therefore, has been to first extract the total base-collector capacitance [6].

A complete analysis for the T-topology is presented. The T-topology, being directly related to device physics, allows checking of the physical relevance of the extracted parameters, and hence is not only useful for circuit design, but also for device optimisation and technology development.

The hybrid-$\pi$ model is included for completeness. Even though it does not represent device physics directly, i.e. the elements $R_\pi$, $C_\pi$, $G_{m0}$, and $\tau$ exhibit frequency dependence especially in the millimeterwave range, this model is the small-signal equivalent of the popular Gummel-Poon HBT large-signal model [12].

## II. Analysis

The HBT T-topology equivalent circuit is shown in Fig.1, with the intrinsic bias-dependent part shown within the dashed box. The internal T-network comprising $R_{bi}$, $R_{be}$, $C_{be}$, $R_{bc}$, $C_{bci}$ and $\alpha$ can be expressed in Z-parameters as follows

$$[Z_{int}] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} R_{bi} + \frac{1}{Y_{be}} & \frac{1}{Y_{be}} \\ \frac{1}{Y_{be}} - \frac{\alpha}{Y_{bc}} & \frac{1}{Y_{be}} + \frac{1}{Y_{bc}}(1-\alpha) \end{bmatrix} \quad (1)$$

where

$$Y_{be} = \frac{1}{R_{be}} + j\omega C_{be} \quad (2)$$

$$Y_{bc} = \frac{1}{R_{bc}} + j\omega C_{bci} \quad (3)$$



0-7803-7239-5/02/$10.00 © 2002 IEEE

and

$$\alpha = \frac{\alpha_0 e^{-j\omega\tau_1}}{1 + j\omega\tau_2} \tag{4}$$

$\alpha$ is the common-base high frequency current gain. $\alpha_0$ is the dc current gain, $\tau_1$ models the transist time, whereas $\tau_2$ corresponds to $\omega_\alpha$, the cutoff frequency.

From (1)-(4) it follows that

$$R_{bi} = Z_{11} - Z_{12} \tag{5}$$

$$R_{be} = \frac{1}{\mathrm{Re}(Z_{12})} \tag{6}$$

$$C_{be} = \frac{1}{\omega}\,\mathrm{Im}(\frac{1}{Z_{12}}) \tag{7}$$

$$R_{bc} = \frac{1}{\mathrm{Re}(Z_{22} - Z_{21})} \tag{8}$$

$$C_{bci} = \frac{1}{\omega}\,\mathrm{Im}(\frac{1}{Z_{22} - Z_{21}}) \tag{9}$$

and

$$\alpha = \frac{Z_{12} - Z_{21}}{Z_{22} - Z_{21}} \tag{10}$$

[1]Now consider the Y-parameters of the complete intrinsic circuit, [Y]. De-embedding $C_{bcx}$ gives

$$\begin{aligned}[Y_{int}] &= [Y] - [Y_{bcx}] \\ &= \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} - \begin{bmatrix} j\omega C_{bcx} & -j\omega C_{bcx} \\ -j\omega C_{bcx} & j\omega C_{bcx} \end{bmatrix}\end{aligned}$$

where $[Y_{bcx}]$ corresponds to $C_{bcx}$.

The inverse of $[Y_{int}]$ gives the Z-parameters of the internal T-network as

$$[Z_{int}] = \frac{1}{\Delta Y_{int}} \begin{bmatrix} Y_{22} - j\omega C_{bcx} & -Y_{12} - j\omega C_{bcx} \\ -Y_{21} - j\omega C_{bcx} & Y_{11} - j\omega C_{bcx} \end{bmatrix} \tag{12}$$

where

$\Delta Y_{int} = \Delta Y - j\omega C_{bcx} \sum Y$ with $\Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21}$ and $\sum Y = Y_{11} + Y_{22} + Y_{12} + Y_{21}$

Since (1) and (12) are equal, the intrinsic base resistance $R_{bi}$ (see (5)) can be expressed as

$$R_{bi} = \frac{Y_{12} + Y_{22}}{\Delta Y - j\omega C_{bcx} \sum Y} \tag{13}$$

and, therefore, we can write

[1]Note that $\alpha$ may also be computed using

$$\alpha = \frac{Y_{21} - Y_{12}}{Y_{11} + Y_{21}} \tag{11}$$

where $Y_{ij}$ are the Y-parameters of the complete intrinsic equivalent circuit. Simple algebraic manipulations show that this equation and eqn.10 for $\alpha$ are, infact, completely identical.

$$\mathrm{Im}\left(\frac{Y_{12} + Y_{22}}{\Delta Y - j\omega C_{bcx} \sum Y}\right) = 0 \tag{14}$$

from which $C_{bcx}$ is easily determined as

$$C_{bcx} = \frac{1}{\omega}\frac{\mathrm{Re}(Y_s)\,\mathrm{Im}(\Delta Y) - \mathrm{Im}(Y_s)\,\mathrm{Re}(\Delta Y)}{\mathrm{Re}(Y_s)\,\mathrm{Re}(\sum Y) + \mathrm{Im}(Y_s)\,\mathrm{Im}(\sum Y)} \tag{15}$$

where $Y_s = Y_{12} + Y_{22}$.

The time constants and $\alpha_0$ are calculated as follows. From (4) and (10), taking the reciprocal of the modulus of (4) and squaring both sides of the equation gives

$$\frac{1}{|\alpha(\omega)|^2} = \frac{1}{\alpha_0^2}\,(1 + \tau_2^2\omega^2) \tag{16}$$

from which it is clear that plotting $1/|\alpha(\omega)|^2$ vs. $\omega^2$ should give a linear graph. The $1/|\alpha(\omega)|^2$-intercept gives $1/\alpha_0^2$ and the gradient gives $(\tau_2/\alpha_0)^2$ and hence $\alpha_0$ and $\tau_2$ can be determined. And finally, $\tau_1$ is calculated using

$$\tau_1 = -\frac{1}{\omega}\tan^{-1}\left(\frac{\mathrm{Im}\left[(1 + j\omega\tau_2)\,\alpha\right]}{\mathrm{Re}\left[(1 + j\omega\tau_2)\,\alpha\right]}\right) \tag{17}$$

### A. The Hybrid-$\pi$ Model

A similar analysis yields the same equations for $C_{bcx}$ and $R_{bi}$ for the hydrid-$\pi$ small-signal equivalent circuit. The equations for extracting the other intrinsic equivalent circuit parameters are identical to those given in [11].

## III. Parameter Extraction

The preceeding section has given exact formulae for evaluating the intrinsic HBT. The modeling effort, therefore, reduces to an accurate determination of the device parasitics which are first determined and de-embedded (e.g.[3],[4]). Next, $C_{bcx}$ is computed using (15) and also de-embedded. All the other intrinsic parameters are then calculated using (5)-(10). The transit times and $\alpha_0$ are evaluated using (16) and (17). A key advantage of calculating each individual element at each measured frequency is that the quality of the extraction can also be checked by looking at any frequency dependence exhibited by the elements. The linear graph used for determining $\alpha_0$ and the transist times also assists in further checks on the quality of the extraction.

## IV. Results

The new expression for $C_{bcx}$ was first verified using synthetic data. The next verification step was tests on actual device data. Here, the new formulation was used to extract the equivalent-circuit parameters of GaInP/GaAs HBT's fabricated at the Ferdinand-Braun-Institut fuer Hoechstfrequenztechnik (FBH, Berlin, Germany) and which are designed for the microwave and lower millimeterwave range, and the parameters of InP/GaInAs HBT's (emitter area 1x10 $\mu m^2$) fabricated in-house at the Technion and which are designed for the millimeterwave range ($f_t/f_{max}$ =

150/200 GHz). The parasitic elements were determined using open-collector measurements. First extraction results show good agreement between measured and modeled parameters for the two different technologies, and give insight to any frequency variations of the intrinsic elements. As an example, Fig.3 shows the frequency (in)dependence of the outer base-collector capacitance for the InP/GaInAs HBT over the entire measured frequency range. The values of $C_{bcx}$ and $C_{bci}$ agree well with the expected geometric ratio of the emitter mesa area to the (base mesa area - emitter mesa area) (see Table 2). S-parameter fits for both the InP- and GaAs-HBTs are shown in Figs.4 and 5, respectively. Table 1 shows the extracted equivalent circuit elements.

## V. Conclusion

Exact equations for modeling of the intrinsic HBT have been presented. Small-signal modeling of HBTs is reduced to an accurate determination of the parasitic element values. First extraction results show that the new formulation can be used for reliable and physically meaningful modeling of HBTs.

*Acknowledgement 1:* The authors wish to thank Dr.-Ing. Matthias Rudoph of the FBH-Berlin for providing GaAs HBT S-parameter data and for useful discussions on HBT modeling.

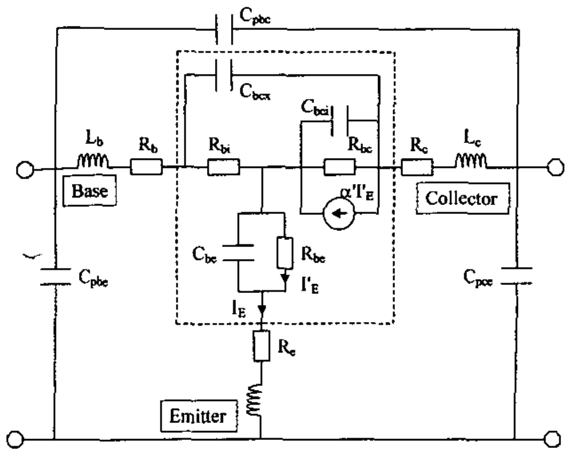


Fig. 1. Small-signal equivalent circuit of the HBT. The dashed box denotes the intrinsic bias-dependent part.

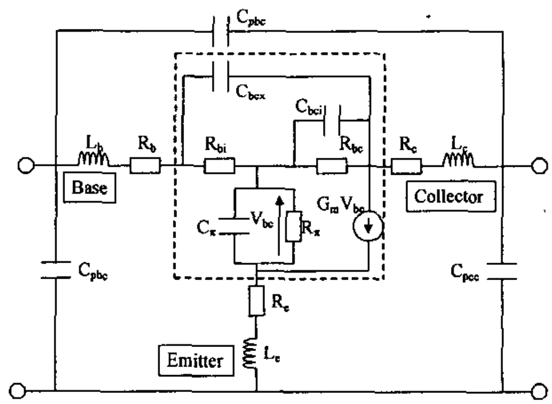


Fig. 2. Hybrid-$\pi$ small-signal equivalent-circuit of the HBT. The dashed box denotes the intrinsic bias-dependent part.

### References


[1] M. Rudolf, R. Doerner, and P. Heymann, "Direct Extraction of HBT Equivalent-Circuit Elements," *IEEE Trans. Microwave Theory Tech.*, vol.47, no.1, pp.82-84, January 1999.

[2] G. Dambrine, A. Cappy, F.Heliodore, and E. Playez, "A new method for determining the FET small-signal equivalent circuit," *IEEE Trans. Microwave Theory Tech.*, vol.36, no.6, pp.1151-1159, July 1988.

[3] Y. Gobert, P.J. Tasker, and K.H. Bachem, " A physical, Yet Simple, Small-Signal Equivalent Circuit for the Heterojunction Bipolar Transistor," *IEEE Trans. Microwave Theory Tech.*, vol.45, no.1, pp.149-153, January 1997.

[4] S. A. Maas and D. Tait, "Parameter Extraction Method for Heterojunction Bipolar Transistors," IEEE Microwave and Guided Wave Letters, Vol.2, No.12, pp.502-504, December 1992.

[5] C.-J. Wei and J.C.M. Hwang, "Direct Extraction of Equivalent Circuit Paramters for Heterojunction Bipolar Transistors," *IEEE Trans. Microwave Theory Tech.*, vol.43, no.9, pp.2035-2040, September 1995.

[6] S.J. Spiegel, D. Ritter, R.A. Hamm, A. Feygenson, and P.R. Smith, "Extraction of the InP/GaInAs Heterojunction Bipolar Transistor Small-signal Equivalent Circuit," *IEEE Trans. Microwave Theory Tech.*, vol.42, no.6, pp.1059-1064, June 1995.

[7] M. Sotoodeh et al., "Stepping Toward Standard Methods of Small-Signal Parameter Extraction for HBTs," *IEEE Trans. Microwave Theory Tech.*, vol.47, no.6, pp.1139-1151, June 2000.

[8] B. Li, S. Prasad, L.-W. Yang, and S.C. Wang, "A Semianalytical Parameter-Extraction Procedure for HBT Equivalent Circuit," *IEEE Trans. Microwave Theory Tech.*, vol.46, no.10, pp.1427-1435, October 1998.

[9] U. Schaper and B. Holzapfl, "Analytical Parameter Extraction of the HBT Equivalent Circuit with T-Like Topology from Measured S-Parameters," *IEEE Trans. Microwave Theory Tech.*, vol.43, no.3, pp.493-498, March 1995.

[10] D.R. Pehlke and D. Pavlidis, "Evaluation of the Factors Determining HBT High-Frequency Performance by Direct Analysis of S-Parameter Data," *IEEE Trans. Microwave Theory Tech.*, vol.40, no.12, pp.1139-1151, December 1992.

[11] D. Costa, W. U. Liu, and J. S. Harris, "Direct Extraction of the AlGaAs/GaAs Heterojunction Bipolar Transistor Small-Signal Equivalent Circuit," *IEEE Trans. on Electron Devices*, Vol. 38, No.9, pp.2018-2024, September 1991.

[12] D. A. Teeter and W. R. Curtice, "Comparison of Hybrid Pi and Tee HBT Circuit Topologies and their Relationship to Large Signal Modeling," *IEEE MTT-S digest*, pp.375-378, 1997.

[13] T. S. Horng, J. M. Hu, and H. H. Huang, "An Extrinsic-Inductance Independent Approach for Direct Extraction of HBT Intrinsic Circuit Parameters," *IEEE MTT-S digest*, 2001.

[14] Y. Suh, E. Seok, J.-H. Shin, B. Kim, D. Heo, A. Raghavan, and J. Laskar, "Direct Extraction Method for Internal Equivalent Circuit Parameters of the HBT Small-signal Hybrid-$\pi$ Model," *IEEE MTT-S digest*, 2000.

[15] S. Bousnina, P. Mandeville, A. B. Kouki, R. Surridge, and F. M. Ghannouchi, "A New Analytical and Broadband Method for Determining the HBT Small-signal Model Parameters," *IEEE MTT-S digest*, 2000.

[16] S. Bousnina, F. M. Ghannouchi, and R. Surridge, "A Novel Direct Extraction Method for Internal Equivalent Circuit Parameters of HBT Small-signal Hybrid-Pi Model," European Microwave Conference Proceedings, 2000.

| | InP/GaInAs | GaInP/GaAs |
|---|---|---|
| | $V_{CE}$ =1.2V, $I_C$=7.8mA | $V_{CE}$ =3V, $I_C$=18mA |
| $C_{pbe}$ (fF) | 31 | 30 |
| $C_{pbc}$ (fF) | 2.8 | 1.0 |
| $C_{pce}$ (fF) | 32.5 | 31.4 |
| $L_b$ (pH) | 31.4 | 38.14 |
| $L_c$ (pH) | 34.8 | 31.32 |
| $L_e$ (pH) | 8.9 | 1.0 |
| $R_b$ ($\Omega$) | 2.3 | 0.92 |
| $R_c$ ($\Omega$) | 0.4 | 0.84 |
| $R_e$ ($\Omega$) | 9.5 | 3.32 |
| $C_{bcx}$ (fF) | 32.5 | 71.5 |
| $R_{bi}$ ($\Omega$) | 20 | 4.0 |
| $R_{be}$ ($\Omega$) | 3.6 | 4.2 |
| $C_{be}$ (fF) | 165 | 353 |
| $R_{bc}$ (k$\Omega$) | 31 | 28 |
| $C_{bci}$ (fF) | 4.6 | 7.5 |
| $\tau_1$ (pS) | 0 | 2.24 |
| $\tau_2$ (pS) | 0.79 | 3.25 |
| $\alpha_0$ | 0.9414 | 0.9896 |
| $f_t$ (GHz) | `123 | 41 |

TABLE I

EXTRACTED EQUIVALENT-CIRCUIT ELEMENTS FOR THE INP/GAINAS- AND GAINP/GAAS-HBTS

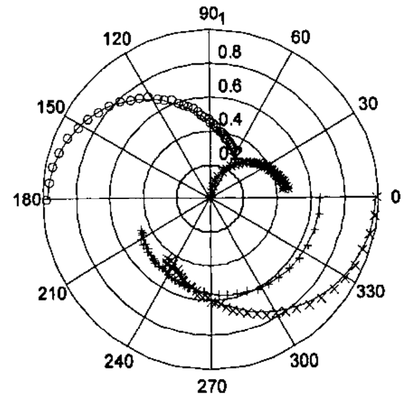


Fig. 4. Comparison of measured (symbols) and calculated (solid lines) S-parameters from 50MHz-40GHz (InP/GaInAs-HBT, $V_{CE}$=1.2V, $I_C$=7.8mA). $S_{11}$ '+', $0.18S_{21}$ 'o', $2S_{12}$ '*', $S_{22}$ 'x'.

| Device Dimensions | $C_{bcx}$ | $C_{bci}$ |
|---|---|---|
| emitter, base mesa | (fF) | (fF) |
| 3x10, 7x16.5 um$^2$ | 33 | 11.5 |
| 4x10, 8x16.5 um$^2$ | 37.5 | 15.0 |

TABLE II

DEVICE GEOMETRY, EXTRACTED EXTRINSIC AND INTRINSIC BASE-COLLECTOR CAPACITANCES FOR INP/GAINAS IN THE ACTIVE REGION

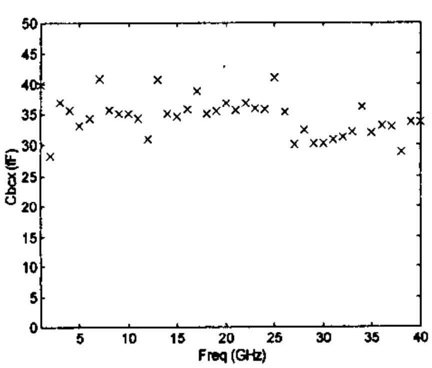


Fig. 3. Extracted $C_{bcx}$ as a function of frequency for the InP/GaInAs HBT

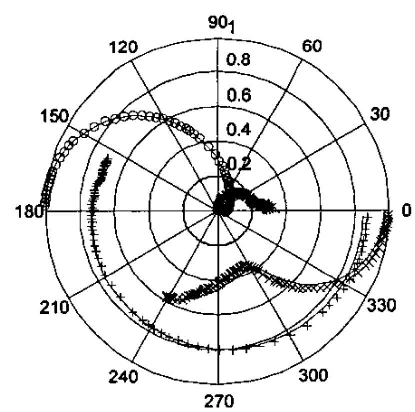


Fig. 5. Comparison of measured (symbols) and calculated (solid lines) S-parameters from 50MHz-50GHz (GaInP/GaAs-HBT, $V_{CE}$=3V, $I_C$=18mA). $S_{11}$ '+', $0.084S_{21}$ 'o', $2S_{12}$ '*', $S_{22}$ 'x'.